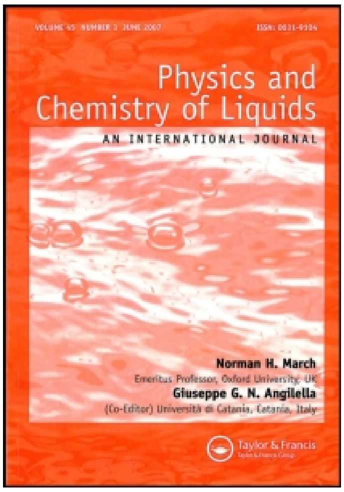

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UTILITY OF COUPLING PARAMETER Γ IN CHARACTERIZING ELECTRICAL CONDUCTIVITY OF PLASMAS: ESPECIALLY HYDROGEN AND COPPER

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Recent experiments of DeSilva and Kunze on the electrical conductivity σ of a copper plasma have been expressed in terms of the plasma coupling parameter $\Gamma = \{(ze)^2/r_s\}/k_BT$. Here r_s measures the mean interparticle spacing, while z denotes the (effective) valency.

Theoretical results for a structureless hydrogen plasma are presented therefore in a plot of ln σ vs Γ. Since the theory is based on Debye shielding, it is valid for Γ less than 1. Curves of ln σ vs Γ vary with density n for these small values of Γ, but the shapes are very similar for each of the three densities considered. At constant Γ, ln σ vs ln n plots are linear for $\Gamma < 1$.

Brief comparison with the values of Cu plasma, both experiment and the theory of Likalter for Cu near the metal-insulator transition, is finally made.

KEY WORDS: Liquid structure factor, Debye screening length, Einstein relation.

1. INTRODUCTION

Following the early experiments of Pantikow and Steinberger[1] which measured the electrical conductivity of a hydrogen plasma up to a temperature of 24,000K, at which the plasma is fully ionized, Leung and March[2] made a theoretical study of this problem. Their work was based on the three assumptions that:

(i) Short-range order of the protons could be neglected: that is the 'liquid' structure factor $S(k)$ could be replaced by unity
(ii) The electron–proton interaction could be treated as weak, under the extreme experimental conditions and
(iii) This interaction, for a single proton, could be modelled by an attractive screened Coulomb potential $(-e^2/r)\exp(-r/1_D)$ where l_D is the Debye screening length given by

$$l_D = (k_BT/4\pi e^2 n)^{1/2} \tag{1}$$

with n the number of electrons per unit volume.

Leung and March[2] pointed out that this latter screened Coulomb form of the potential is valid when the thermal energy k_BT is greater than the mean potential energy e^2/r_S: that is

$$\Gamma = \frac{e^2}{r_s k_B T} < 1 \tag{2}$$

where Γ is the usual plasma coupling parameter. The comparison of this theoretical work with experiment revealed excellent agreement[2] with the results of Plantikow and Steinberger[1].

Interest in this type of study has been revived by the very recent experiments of DeSilva and Kunze[3]. These workers presented data on the electrical conductivity of strongly coupled copper plasmas, in the range of densities from 0.3–3 gm/cm^3, for temperatures ranging from 8000 to 30,000 K. Though these workers compared their measurements with a variety of theoretical models, some of which were successful at a semi-quantitative level, the conclusion of immediate interest for the present study is that their data for the conductivity $\sigma(n, T)$, when taken to large values of Γ, became essentially of the 'universal' form

$$\sigma = \sigma(\Gamma); \quad \text{large } \Gamma \text{ limit.} \tag{3}$$

This is the motivation for the present work on the utility of the coupling parameter Γ in characterizing the electrical conductivity of plasmas. In section 2 immediately below we therefore return to the case of a hydrogen plasma, while in section 3 both experiment and a theory of Likalter will be referred to again briefly for Cu.

2. ELECTRICAL CONDUCTIVITY IN TERMS OF Γ FOR STRUCTURELESS HYDROGEN PLASMA

We turn then, in somewhat more detail, to consider results for a fully ionized hydrogen plasma in the regime $\Gamma < 1$ appropriate to the experiments of Plantikov and Steinberger[1]. The basic assumptions underlying the early work of Leung and March[2] were set out in section 1. Then, applying Boltzmann transport theory, one is led to the result, for single-centre scattering from the screened Coulomb potential

$$V(r) = -(e^2/r)\exp(-r/1_D) \tag{4}$$

with the Debye length given in terms of the temperature T and the density n in eqn (1):

$$\sigma = \frac{e^2}{3\pi^2 m}\int_0^\infty dE\, \frac{\partial f}{\partial E}\, k^3\, \tau(E) \tag{5}$$

where f is the Fermi distribution function, given in terms of the chemical potential μ by

$$f(E)=\frac{1}{1+\exp\left(\frac{E-\mu}{k_BT}\right)}. \tag{6}$$

In the model of Leung and March[2], the relaxation time $\tau(E)$ as a function of energy E where

$$E=\frac{\hbar^2 k^2}{2m} \tag{7}$$

is given by

$$\frac{1}{\tau(E)}=\frac{4\Omega mk}{\pi\hbar^3}\int_0^1 ds\, s^3\,|\tilde{V}(2ks)|^2. \tag{8}$$

For the screened Coulomb form (4), the fourier transform $\tilde{V}(k)$ yields

$$\tilde{V}(2ks)=\frac{-4\pi e^2}{(2ks)^2+1_D{}^{-2}}\cdot \tag{9}$$

Inserting this in eqn (8) the integration can be performed to obtain

$$\frac{1}{\tau(E)}=\frac{4\Omega \mathrm{m}e^2\pi}{\hbar^{3k}k^3}\left[\frac{1}{2}\ln\left\{\frac{\alpha^2+1}{\alpha^2}\right\}\frac{-1}{2}+\frac{1}{2}\left\{\frac{\alpha^2}{\alpha^2+1}\right\}\right] \tag{10}$$

where Ω is the system volume while

$$\alpha=(2kl_D)^{-1}. \tag{11}$$

Finally, inserting eqn (10) into eqn (5) enables the conductivity of the hydrogen plasma to be calculated as a function of n and T. The chemical potential μ in units of the thermal energy k_BT obtained by the above route is plotted in Figure 1 as a function of the coupling parameter Γ for three densities, namely $n=10^{17}, 10^{19}$ and 10^{21} cm^{-3}.

Our next objective is to study the way the electrical conductivity σ in the above model, valid for $\Gamma<1$, depends on Γ, and to this end Figure 2 shows plots of ln σ and Γ for the three values of n quoted above. Evidently, though there is appreciable 'residual' dependence on the density n, the shapes of the three curves in Figure 2 are very similar. This has prompted us to consider then the variation of ln σ with density at constant Γ and the results thus obtained for two values $\Gamma=0.04$ and 0.35 are displayed in Figure 3. It will be seen that the ln-ln plots are then linear, with almost identical slopes for the two values of Γ chosen.

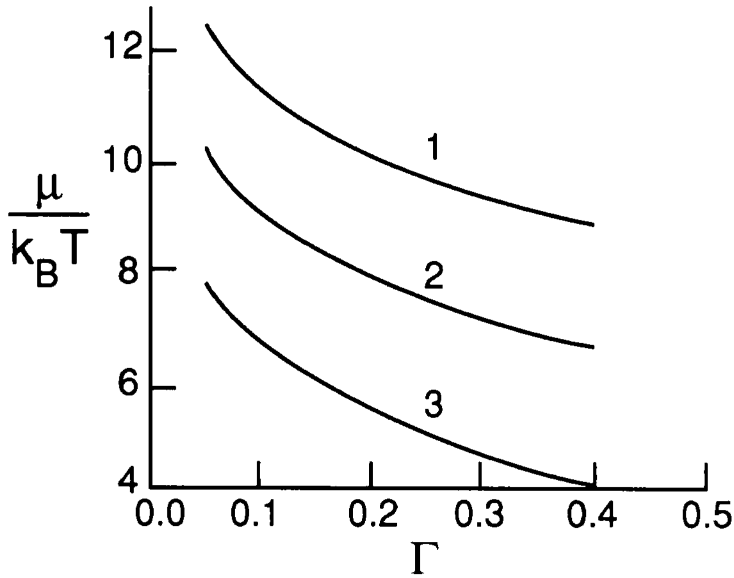

Figure 1 Chemical potential μ in units of thermal energy k_BT for hydrogen plasma. Curves 1–3 are for the three densities considered; namely $n = 10^{17}, 10^{19}$ and 10^{21} cm^{-3}. Independent variable is the plasma coupling constant Γ in eqn (2).

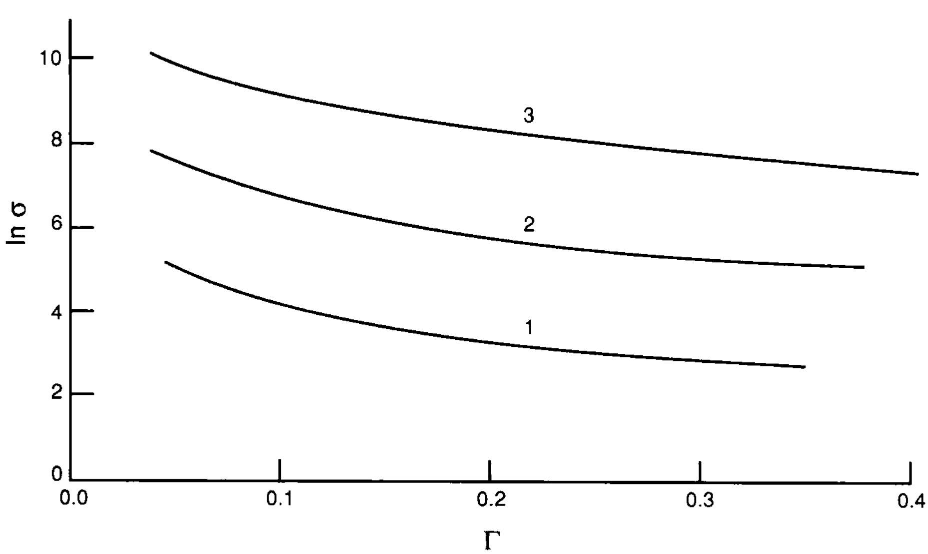

Figure 2 Shows ln plot of electrical conductivity σ versus plasma coupling parameter Γ. Curves 1–3 as in Fig. 1.

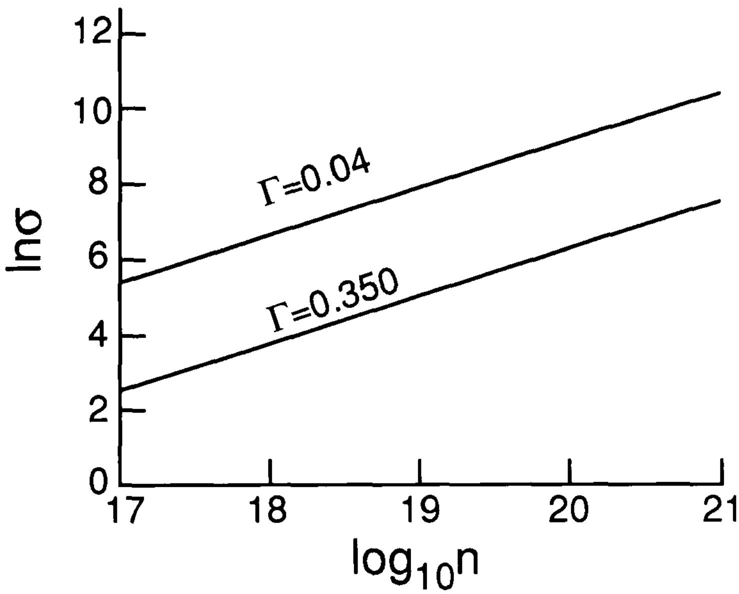

Figure 3 Shows ln plot of electrical conductivity σ as a function of log of density n, for constant Γ. (values taken as 0.04 and 0.35).

Following these results on the hydrogen plasma, let us now return to the study of copper plasmas.

3. ELECTRICAL CONDUCTIVITY VS Γ FOR COPPER PLASMAS

In Fig 10(a) of their paper, DeSilva and Kunze[3] (ref 3) have plotted log σ vs log Γ for copper plasmas. As they emphasize, it is striking that, for $\Gamma > 10$, the conductivity data converge on to a line, strongly suggesting that for sufficiently large Γ the electrical conductivity will become a function solely of this one parameter Γ. For $\Gamma < 10$, there is clearly 'residual' dependence on the density n, as is already evident in Figure 2 above for the case of hydrogen plasmas.

It is of interest here to note, in the above context, that in Figure 8 of ref 3, a comparison has been made between the measurements of conductivity of Cu plasmas as a function of ln n with the theory of Likalter[4]. We shall therefore look a little more closely at this, admittedly primitive, theory, but now expressed in terms of the coupling parameter Γ.

Essentially, Likalter [4]was concerned with the liquid-vapour critical point in relation to the metal-non-metal transition. Therefore, he calculated the electrical conductivity σ using arguments near the percolation threshold. The argument goes that, in this regime, the valence electrons of copper are in 'nearly-localized atomic-like states', diffusing through random walks between neighbouring atoms. In this picture, one can write the diffusion constant D as

$$D=\frac{R^2\theta}{3\tau} \tag{12}$$

where R is the mean interatomic distance, τ is the 'time of flight' given by $\tau = R/v_T$ where v_T is the mean thermal velocity. Finally the factor θ represents an attempt to characterize the 'partial localization' of the valence electrons.

Applying then the Einstein relation, Likalter [4]writes

$$\sigma = e^2 n_e \tau\theta/m : n_e = z n_i \tag{13}$$

with n_i the density of ions. Figure 3 of Likalter's paper is a plot of ln σ versus density at the critical temperature $T_c \sim 7600\,\text{K}$ of Cu. As he notes, in a wide density range from the normal value at melting to the transition point, the conductivity varies relatively slowly, but it rapidly decreases below this point.

Now let us turn to consider briefly the form of θ given by Likalter[4] on the 'non-metallic' side of the transition. This can be written as

$$\theta \simeq [2(\Delta_1 T)^{1/2}/\pi^{1/2}(\Delta_2-\Delta_1)]\ \exp(-\Delta_1/T){:}\ \Delta_2 > \Delta_1 \gg T. \tag{14}$$

Here the excitation energy Δ_1 corresponding to the percolation threshold and that corresponding to 'nearly -free' motion are given by Likalter as

$$\Delta_i = I - e^2(4\pi n_a/3\zeta_i)^{1/3} \quad (i=1,2) \tag{15}$$

where $\zeta_1 \simeq 0.3$ and $\zeta_2 \simeq 0.75$, with n_a the atomic density.

Inserting this into eqn (14) one finds

$$\theta \simeq \exp(-I/T)\ \exp(c\Gamma), \tag{16}$$

I being the ionization potential. Though Likalter's model, as already discussed, is still primitive (for a different approach via the Einstein relation, see March and Tosi[5]) and one should not expect the details to be quantitative, one can see from the above how ln $\sigma = \ln(ne^2\tau/m) + \ln\theta$ could become dominated by Γ, though in the detailed form of Likalter's present model the dependence on ionization potential I is too strong to be in accord with the large Γ findings of DeSilva and Kunze[3].

4. SUMMARY

By comparing and contrasting hydrogen and copper plasmas, it is clear that it is revealing in both cases to express the electrical conductivity in terms of the plasma coupling parameter Γ. In the former case, the experiments of Plantikow and

Steinberger [1] are in the regime of small Γ, where both experiment and theory for this system show that residual density dependence then remains. Boltzmann transport, with structureless ions, is adequate for a quantitative explanation[2] of the experimental data in this regime.

However, for copper plasmas, the measurements of DeSilva and Kunze[3] have covered a wide range of coupling parameter Γ and have clearly demonstrated that, for $\Gamma > 10$, $\sigma = \sigma(\Gamma)$, any residual density dependence having by then been largely suppressed. This appears to require a quite different treatment from that of Boltzmann transport theory and the percolation arguments of Likalter[4] are cited as a possible future starting point of a transport theory in which now partially localized valence electrons of Cu in somewhat atomic-like states can perform diffusive motions between neighbouring atoms.

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